

Quantum Otto cycle efficiency on coupled qudits

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Abstract

Properties of the coupled particles with spin $3/2$ (quartits) in a constant magnetic field, as a working substance in the quantum Otto cycle of the heat engine, are considered. It is shown that this system as a converter of heat energy in work (i) shows the efficiency 1 at the negative absolute temperatures of heat baths, (ii) at the temperatures of the opposite sign the efficiency approaches to 1, (iii) at the positive temperatures of heat baths antiferromagnetic interaction raises efficiency threefold in comparison with uncoupled particles.

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I. INTRODUCTION

As it is known, thermodynamics has the broadest applications for description of many physical phenomena [1, 2]. The quantum thermodynamics studies dynamics of heat and work in quantum systems. Researchers began to study quantum thermodynamic engines after appearance of works [3, 4]. Thermodynamic cycles can be reformulated for quantum systems [5–14]. One of the important quantum thermodynamic cycles is an Otto cycle. Similarly to a classical Otto cycle the quantum Otto cycle consists of two isochoric and two adiabatic stages too. A quantum isochoric process corresponds to a thermal exchange between a working body and thermal baths. During the quantum isochoric process only population of levels is reconstructed, whereas at the adiabatic process the working body produces work at the expense of power level changes. The adiabatic process can be thermodynamic adiabatic or quantum adiabatic. A process is thermodynamic adiabatic if the working substance is thermally isolated from a heat bath. However it does not exclude transitions of purely quantum nature between levels while at the quantum adiabatic process the population density of levels is fixed.

The coupled spin systems can be used as quantum thermodynamic engines. In small systems with a finite number of degrees of freedom as finite-dimensional effects and quantum effects essentially influence thermodynamic properties of the system. The aim of this work is a research of quantum systems with a finite number of levels as a working substance in a quantum Otto cycle, including viewing of the negative absolute temperatures [15–17]. A particle with spin-3/2 was studied both in the thermodynamic description in a stationary case in [15] and in finite-time quantum thermodynamics in [18]. The two particles coupled by Heisenberg exchange interaction one of them with spin-3/2 and another with spin-1/2 in an Otto cycle are investigated in the work [19].

The article is organised as follows. In section II, the properties of a working substance consisting of two coupled spins 3/2 in a static magnetic field are described. In section III, a quantum heat Otto cycle is presented. Section IV presents the results graphically at the concrete control parameters. The final section V summarises the findings. In the Appendix auxiliary analytical formulae are presented.

II. WORKING SUBSTANCE

The choice of a working substance for operation of the quantum heat engine is essential [20–22]. The working substance in our case is featured by the Hamiltonian \hat{H} of two coupled spins-3/2 (biquartit) with permutation symmetry of particles and the isotropic exchange interaction in a external static magnetic field h applied along the z-axis:

$$\hat{H} = \mu h (E_{4 \times 4} \otimes S_3 + S_3 \otimes E_{4 \times 4}) + 4J \vec{S} \otimes \vec{S}, \quad (1)$$

in which μ is the quartit magnetic moment. The external control Hamiltonian commutes with the internal interaction. $E_{4 \times 4}$ is the identity matrix, S_1, S_2, S_3 is the matrix representation of components of the spin 3/2 (A1), J is the interaction constant. Cases $J > 0$ and $J < 0$ correspond to antiferromagnetic and ferromagnetic interactions, respectively.

By the known partition function Z (A2) it is possible to calculate the free energy F , the entropy S , the internal energy U and the heat capacity C :

$$F = -1/\beta \ln Z, S = \beta^2 \partial_\beta F, U = \partial_\beta \beta F, C = -\beta^2 \partial_{\beta\beta} \beta F, \quad (2)$$

where $\beta = 1/k_B T$ is the inverse temperature, Z is the partition function. In the formulae (2) the β partial differentiation indicates that the other parametres are fixed. We use units chosen so that the magnetic moment equals 1, the Boltzmann constant k_B equals 1, hence J, h, T in Joules.

According to [23], [24] the entanglement is determined by the values of decomposition of the density matrix on the basis $\rho \propto \sum_{i=0}^{D-1} \sum_{j=0}^{D-1} R_{ij} C_i \otimes C_j$

$$m_{SM} = \sqrt{1/(D-1) \sum_{i=0}^{D-1} \sum_{j=0}^{D-1} (R_{i0} R_{0j} - R_{ij})^2}, \quad (3)$$

where D is the qudit dimension of basis C_i (for a quartit $D = 16$, for a qubit $D = 4$) and R_{ij} are the components of the Bloch vector and $R_{00} = 1$.

A. Local temperatures

For the Hamiltonian (1) the quartit density matrix $\varrho = \frac{1}{2\sqrt{5}} \sum_{i=0}^{15} R_{i0} C_i$ and a local Hamiltonian is diagonal. The local entropy s and the internal energy u are defined by

formulae

$$s = - \sum_{k=1}^4 \pi_k \ln \pi_k, \quad u = \sum_{k=1}^4 \varepsilon_k \pi_k, \quad (4)$$

where the diagonal elements of the reduced density matrix ϱ look like

$$\pi_1 = 1/20(5 - 5p_1 + p_2 - 5p_3 - 5p_4 - 5p_5 + 5p_6 + 5p_7 + 4p_9 - 4p_{11} + 15p_{12} - 5p_{13} - 5p_{14} - p_{15} + 5p_{16}),$$

$$\pi_2 = 1/20(5 + p_1 + 3p_2 - 5p_3 + 5p_4 - 5p_5 - 5p_6 + 5p_7 - 4p_9 + 4p_{11} - 5p_{12} - 5p_{13} - p_{14} + 7p_{15} + 5p_{16}),$$

$$\pi_3 = 1/20(5 + 3p_1 + p_2 + 5p_3 - 5p_4 - 5p_5 + 5p_6 - 5p_7 - 4p_9 + 4p_{11} - 5p_{12} + 5p_{13} + 7p_{14} - p_{15} - 5p_{16}),$$

$$\pi_4 = 1/20(5 + p_1 - 5p_2 + 5p_3 + 5p_4 + 15p_5 - 5p_6 - 5p_7 + 4p_9 - 4p_{11} - 5p_{12} + 5p_{13} - p_{14} - 5p_{15} - 5p_{16}),$$

and π_i are the populations of the reduced (local) diagonal quartit matrix (it was provided with a choice of the Hamiltonian (1)). And only for the diagonal matrix it is possible to determine the local entropy correctly. The eigenvalues ε_k of a local quartit Hamiltonian are equal $\hbar/2(3, 1, -1, -3)$. Formulae of level populations p_i are given in the Appendix. The local quartit temperature is equal

$$\beta_{loc} = \frac{1}{T_{loc}} = \frac{\partial s}{\partial u} = \frac{\partial s / \partial \beta}{\partial u / \partial \beta}. \quad (5)$$

The local temperature is not equal to the system temperature of two coupled quartits $\beta = 1/T$ [21]. We define the inverse spectroscopic temperature as [25]:

$$\beta_{Mloc} = - \left(1 - \frac{\pi_1 + \pi_M}{2} \right)^{-1} \sum_{i=2}^M \left(\frac{\pi_i + \pi_{i-1}}{2} \right) \left(\frac{\ln \pi_i - \ln \pi_{i-1}}{\varepsilon_i - \varepsilon_{i-1}} \right), \quad (6)$$

where π_i is the probability to find the quantum system at the energy ε_i , M is the number of the highest energy level ε_M , while the lowest one is labelled ε_1 . Actually it is a definition of the ensemble average of a random quantity $-\frac{\ln \pi_i - \ln \pi_{i-1}}{\varepsilon_i - \varepsilon_{i-1}}$ with the distribution function density $\left(1 - \frac{\pi_1 + \pi_M}{2} \right)^{-1} \left(\frac{\pi_i + \pi_{i-1}}{2} \right)$.

We shall compare this expression for the local temperature with the temperature definition (5) in section IV.

III. HEAT OTTO CYCLE

We describe a working substance with a Gibbs quantum equilibrium distribution. The working substance passes through 4 stages of the Otto cycle. Therefore in the theoretical

description of this Gedankenexperiment the temperature of baths and the temperature of the working substance are equal because of the assumption about quasi-stationarity and adiabaticity. The power as a done work for infinite time is equal to zero. In the time-finite description at nonadiabaticity the efficiency is less, but the power produced will be distinct from zero.

If to consider the equation for a density matrix of the working substance $\partial_t \rho = -i[\hat{H}, \rho] + L\rho$, where a Lindblad operator L takes into account the environmental influence, then in a stationary case $\partial_t \rho = 0$, and the environmental influence means that for a working substance temperature T (in the first stage) or T' (in the third stage) is established. In the assumption of a weak environmental influence of the term $L\rho$ is little and we obtain the equation $[\hat{H}, \rho] = 0$ for a density matrix ρ . The solution for the density matrix is any function depending on the Hamiltonian \hat{H}/T , where T is the temperature. The Gibbs quantum distribution is followed from the requirement $\text{Tr}\rho = 1$.

Let's feature 4 stages of a quantum quasi-static Otto cycle [20].

Stage 1: the system of two coupled quartits in a magnetic field h attains thermodynamic equilibrium with a heat bath of temperature T . The occupation probabilities are determined by temperature T and a magnetic field h . Thus the occupations change, and the energy levels do not change. The work is not produced during this isochoric process, and the working substance absorbs heat Q_1 from the bath:

$$Q_1 = \sum_{i=1}^{16} e_i(p_i - p'_i). \quad (7)$$

Stage 2: the system is isolated from the heat bath and the magnetic field is changed from h to h' by an adiabatic process, and the energy levels slowly change. According to the adiabatic theorem the occupation probabilities of each energy level maintain. The work is produced:

$$W_2 = \sum_{i=1}^{16} p_i(e'_i - e_i). \quad (8)$$

Stage 3: the system is brought in contact with a heat bath at temperature T' . Upon attaining thermodynamic equilibrium with the bath the occupation probabilities are determined by temperature $T' < T$ and a magnetic field h' . The system gives off heat energy Q_3 to the bath:

$$Q_3 = \sum_{i=1}^{16} e'_i(p'_i - p_i). \quad (9)$$

Stage 4: the system is removed from a cold bath and undergoes another adiabatic process which changes the magnetic field from h' to h but keeps the occupation probabilities unaffected. The energy levels slowly change and the work W_4 is produced:

$$W_4 = \sum_{i=1}^{16} p'_i(e_i - e'_i). \quad (10)$$

The system is brought in contact with a heat bath at temperature T . Heat is absorbed from the bath and the system returns to its initial state.

Note that $Q \geq 0$ means that heat is absorbed from the bath by the system, and $W \geq 0$ means that work is done on the system and the opposite for the opposite direction of the inequalities.

At non-adiabatic transitions fast dynamics on adiabatic branches is responsible for frictional losses. As a result the system is incapable to follow adiabatically along time-dependent changes of the Hamiltonian. The deviation from the quantum adiabatic behavior is expressed by losses which appear from generation of inertial components on adiabatic curves and their dephasing on isochores [26]. All the cycle is presented on the diagramme (11).

$$\begin{array}{ccc} 1 & \xrightarrow{Q_1} & 2 \\ W_4 \uparrow & & \downarrow W_2 \\ 4 & \xleftarrow{Q_3} & 3 \end{array} \quad (11)$$

The energy change during the cycle is equal to zero:

$$Q_1 + W_2 + Q_3 + W_4 = 0. \quad (12)$$

The heat transferred in *Stage 1* and in *Stage 3* respectively is

$$Q_1 = \sum_{i=1}^{16} e_i(p_i - p'_i) = Jm + hn, \quad Q_3 = \sum_{i=1}^{16} e'_i(p'_i - p_i) = -Jm - h'n, \quad (13)$$

where

$$\begin{aligned} m = & -11(p_1 - p'_1 + p_2 - p'_2 + p_9 - p'_9) + \\ & 9(p_5 - p'_5 + p_{11} - p'_{11} + p_{12} - p'_{12} + p_{13} - p'_{13} + p_{14} - p'_{14} + p_{15} - p'_{15} + p_{16} - p'_{16}) - \\ & 3(p_3 - p'_3 + p_4 - p'_4 + p_6 - p'_6 + p_7 - p'_7 + p_{10} - p'_{10}) - 15(p_8 - p'_8), \end{aligned} \quad (14a)$$

$$\begin{aligned} n = & -p_1 + p'_1 + p_2 - p'_2 - p_4 + p'_4 + p_6 - p'_6 - p_{14} + p'_{14} + p_{15} - p'_{15} + \\ & 2(-p_3 + p'_3 + p_7 - p'_7 - p_{13} + p'_{13} + p_{16} - p'_{16}) + 3(-p_5 + p'_5 + p_{12} - p'_{12}) \end{aligned} \quad (14b)$$

The work is done in Stage 2 and Stage 4 when the energy levels change at the fixed occupation probabilities. Due to energy level changes the work done by the quantum heat engine is

$$W_{out} = W_2 + W_4 = (h' - h)n, \quad (15)$$

where

$$W_2 = (h - h') (p_1 - p_2 + 2p_3 + p_4 + 3p_5 - p_6 - 2p_7 - 3p_{12} + 2p_{13} + p_{14} - p_{15} - 2p_{16}), \quad (16)$$

$$W_4 = (h' - h) (p'_1 - p'_2 + 2p'_3 + p'_4 + 3p'_5 - p'_6 - 2p'_7 - 3p'_{12} + 2p'_{13} + p'_{14} - p'_{15} - 2p'_{16}). \quad (17)$$

The efficiency of transformation of heat into work at $Q_1 > 0$, $Q_3 < 0$ is

$$\eta = \frac{W_{out}}{Q_{in}} = \frac{-(W_2 + W_4)}{Q_1} = \frac{\eta_0}{1 + \frac{Jm}{hn}}. \quad (18)$$

It is obvious that the interaction between particles can give both the enhancement and the reduction of the efficiency concerning noninteracting particles. For uncoupled particles that is $J = 0$ the efficiency is $\eta_0 = 1 - \frac{h'}{h}$ [27].

A. Local description

In this subsection, following the article [20], we feature how the individual quartits undergo the cycle. Heat, transferred locally between one quartit and a heat bath, is

$$q_1 = \frac{h}{2}n, \quad q_2 = -\frac{h'}{2}n, \quad (19)$$

for hot and cold baths accordingly. The work done by one particle is

$$w = q_1 + q_2 = \frac{h - h'}{2}n. \quad (20)$$

$$W = 2w = (h - h')n. \quad (21)$$

Thus the total performed work is the sum of local work obtained from each qudit. It is a consequence of permutation symmetry of the hamiltonian.

The total heat, absorbed (produced) by the system in *Stage 1* (*Stage 3*) can be written as

$$Q_1 = Jm + 2q_1, \quad Q_2 = -Jm - 2q_2. \quad (22)$$

IV. RESULTS

We illustrate analytical results graphically at the control parameters (h, h', J, T, T') . These parameters characterise the working substance at the different stages.

Working substance. Fig. 1 shows the entropy dependence on the internal energy. Dependences of the internal energy, the entropy and the heat capacity on inverse temperature are shown in Fig. 2. Coupling of spins breaks the symmetry which is at the equidistant disposition of energy levels in the system [15]. Dependence of the entanglement on the heat capacity is presented in Fig. 3. The dependence singularity is based on the fact that at a small constant J is multivalued, and at a big one it is two-valued. Numerical comparison of dependence of local inverse temperature definitions from the inverse system temperature is given in Fig. 4. Both definitions give the same results at a small interaction constant. At $J = 0, 0.1, 0.15$ there is the full coincidence of definitions of local temperatures both at negative and positive β (bold lines). At $J > 0.17$ there is an appreciable discrepancy at positive β .

At $J < 0$ the divergence is observed at negative temperatures, that is the graphs are symmetric concerning the origin of coordinates. These divergences both at $J < 0$ and at $J > 0$ are caused by the energy level perturbation.

We describe the efficiency of a quantum Otto cycle on coupled qubits for some possible sets of positive and negative signs of the quantities Q_1, W_2, Q_3, W_4 . Reduced letters in plots show the characteristics of two coupled qubits (biqubit) for comparison with the paper [20]. It is possible due to control parameters h, h', J . It is obvious that at $h = h', J \neq 0, Q_1 > 0, W_2 = 0, Q_3 < 0, W_4 = 0$, that is heat is just transferred from a hot bath into a cold one.

The quantum heat engine between baths with negative absolute temperatures. At negative temperatures of heat baths $T < 0, T' < 0, |T| < |T'|$ the situation, when $Q_1 > 0, Q_3 > 0, W_2 < 0, W_4 < 0$ (see Fig. 5) is possible. In this case, the efficiency of conversion of heat in work is equal to 1 [15], according to (12) and the efficiency definition $\eta = \frac{W_{out}}{Q_{in}} = \frac{-(W_2+W_4)}{Q_1+Q_3} = 1$.

The quantum heat engine between baths with absolute temperatures of the opposite sign. At temperatures of baths $T < 0, T' > 0$ the situation, when $Q_1 > 0, Q_3 < 0, W_2 < 0, W_4 < 0$ (see Fig. 6) is possible. In this case the efficiency of conversion of heat in work is equal

$\eta = \frac{-(W_2+W_4)}{Q_1}$ (18) and because of a small leakage it approaches unity, as shown in Fig. 7.

A shift of the maximum efficiency in the biquartit in regard to the biqubit is observed.

At a modification of driving parameters the efficiency can exceed more than three times the efficiency of uncoupled 3/2-spins, as seen from Fig. 8. For the biqubit the maximum efficiency is moved towards the increase of the interaction constant [20].

The quantum heat engine of conversion of work in heat between baths with the positive temperatures. At the positive temperatures of baths $T > 0$, $T' > 0$, $T > T'$ the situation, when $Q_1 < 0$, $Q_3 < 0$, $W_2 > 0$, $W_4 > 0$ (see Fig. 9) is possible. In this case the efficiency of conversion of work in heat equals $\eta = \frac{-(Q_1+Q_3)}{(W_2+W_4)} = 1$. At some set of parameters Q_1 changes its sign, then the total work $W_2 + W_4$ changes its sign, that is in a neighbourhood $Q_1 = 0$ the work done over the system, is entirely converted in heat [18].

The work as a function of the entanglement in a biquartit. The entanglement m_{SM} and the work W_2, W_4 in the Otto cycle are determined with the matrices $\rho(e_i/T)$, $\rho'(e'_i/T')$ respectively. Fig. 10 shows the parametric dependence of the work as a function of the entanglement. It is evident that the work increases along with the increase of entanglement at $J < 0$ in the second and fourth stages, at $J > 0$ the work decreases with the increase of entanglement. In the absence of interaction the entanglement is equal to zero.

The total work per cycle $-(W_2 + W_4)$ as a function of the magnetic field h' in a biquartit. In the considered approach in limit of small systems with only a few degrees of freedom the necessary condition for a heat engine $h'/T' > h/T$, for a refrigerator $h'/T' < h/T$. In a multilevel system as it was marked in [28], it is difficult to find simple criteria to answer when the Otto cycle is a heat engine, and when it is a refrigerator. At some parameters these criteria (see Fig. 11) are carried out, and at others are not.

For $J = 0$ below the Carnot point (left vertical line, $h'/h = T'/T$) the device acts as a refrigerator and above it until $h' = h$ it performs as an engine. For $h' > h$ the device performs as a heater as it takes work to make the cold bath hotter. At $J < 0$ the done work decreases and the Carnot point slightly moves to the left. At positive $J > 0$ the done work decreases and the Carnot point moves to the right before coincidence with the point $h' = h$ at $J = 0.2$. In this case the device works as a refrigerator ($h' < h$) or as a heater ($h' > h$).

V. CONCLUSION

The quasi-stationary quantum Otto cycle, when the working body is the coupled system of two 3/2-spins, being in a magnetic field, is explored. Some performances of the quantum Otto cycle on the coupled spins, generated by various sets of driving parameters, are considered. The analysis of possible quantities of the cycle efficiency depending on driving parameters is carried out. There are the restrictions on driving parameters $T > T'$, $h'/T' > h/T$ for the conversion of heat in work (see Fig. 5, 6, 7, 8). It is shown that the efficiency of conversion of heat in work at negative temperatures of heat baths equals 1, at temperatures of the opposite sign it approaches 1. At positive temperatures of heat baths the antiferromagnetic interaction of spins [20] raises the efficiency more than threefold in comparison with uncoupled spins [20]. The dependence on a system size is revealed in a displacement of the maximum efficiency in regard to the enhancement of the interaction constant (see Fig. 7, 8).

Dependence of the work on entanglement with the limiting values of efficiency in cases of conversion of heat into work and work into heat is presented.

When dealing with realistic systems, Quantum Thermodynamics introduces finite time in the analysis. For the Carnot, Otto, Stirling and other cycles time is introduced on all stages. The pioneering studies in finite time quantum thermodynamics in the method of quantum generators of open systems were done by R. Kosloff and co-workers in works [5, 6, 29–32]. Nowadays other approaches [18, 33–35] are actively developed. It is necessary also to define more exactly the quantum thermodynamical work and heat in order to study local effective dynamics in microsystems [36, 37].

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Appendix A

The matrix representation of a vector of a spin 3/2 looks like

$$S_1 = \begin{pmatrix} 0 & \frac{\sqrt{3}}{2} & 0 & 0 \\ \frac{\sqrt{3}}{2} & 0 & 1 & 0 \\ 0 & 1 & 0 & \frac{\sqrt{3}}{2} \\ 0 & 0 & \frac{\sqrt{3}}{2} & 0 \end{pmatrix}, S_2 = \begin{pmatrix} 0 & -\frac{i\sqrt{3}}{2} & 0 & 0 \\ \frac{i\sqrt{3}}{2} & 0 & -i & 0 \\ 0 & i & 0 & -\frac{i\sqrt{3}}{2} \\ 0 & 0 & \frac{i\sqrt{3}}{2} & 0 \end{pmatrix}, S_3 = \begin{pmatrix} \frac{3}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & -\frac{3}{2} \end{pmatrix}. \quad (\text{A1})$$

The density matrix on the basis of eigenfunctions of the Hamiltonian (the Gibbs representation) looks like

$$\rho = e^{-\beta \hat{H}} / Z = \sum_{i=1}^{16} p_i P_i, \quad (\text{A2})$$

where $\beta = 1/T$ is the inverse temperature, $Z = \text{Tr } e^{-\beta \hat{H}} = \sum_{i=1}^{16} e^{-\beta e_i}$ is the partition function, $p_i = e^{-\beta e_i} / Z$ are the occupation densities, $P_i = |e_i\rangle \langle e_i|$, $P_i P_k = \delta_{i,k} P_i$ are the projectors constructed of eigenvectors of the Hamiltonian $|e_i\rangle$, corresponding to the eigenvalues $e_i(h, J) \equiv e_i$; $e_1 = -h - 11J$, $e_2 = h - 11J$, $e_3 = -2h - 3J$, $e_4 = -h - 3J$, $e_5 = -3h + 9J$, $e_6 = h - 3J$, $e_7 = 2h - 3J$, $e_8 = -15J$, $e_9 = -11J$, $e_{10} = -3J$, $e_{11} = 9J$, $e_{12} = 3h + 9J$, $e_{13} = -2h + 9J$, $e_{14} = -h + 9J$, $e_{15} = h + 9J$, $e_{16} = 2h + 9J$, $\sum_{i=1}^{16} e_i = 0$, $e'_i(h', J) \equiv e'_i$.

The normalized eigenvectors equal:

$$\begin{aligned} |e_1\rangle &= \sqrt{10/3}(0_7, 1, 0_2, -\frac{2}{\sqrt{3}}, 0_2, 1, 0_2), |e_2\rangle = \sqrt{10/3}(0_2, 1, 0_2, -\frac{2}{\sqrt{3}}, 0_2, 1, 0_7), \\ |e_3\rangle &= 1/\sqrt{2}(0_{11}, -1, 0_2, 1, 0), |e_4\rangle = 1/\sqrt{2}(0_7, -1, 0_5, 1, 0_2), |e_5\rangle = (0_{15}, 1), \\ |e_6\rangle &= 1/\sqrt{2}(0_2, -1, 0_5, 1, 0_7), |e_7\rangle = 1/\sqrt{2}(0, -1, 0_2, 1, 0_{11}), \\ |e_8\rangle &= 1/2(0_3, -1, 0_2, 1, 0_2, -1, 0_2, 1, 0_3), |e_9\rangle = \sqrt{20}/3(0_3, 1, 0_2, -\frac{1}{3}, 0_2, -\frac{1}{3}, 0_2, 1, 0_3), \\ |e_{10}\rangle &= 1/2(0_3, -1, 0_2, -1, 0_2, 1, 0_2, 1, 0_3), |e_{11}\rangle = 1/\sqrt{20}(0_3, 1, 0_2, 3, 0_2, 3, 0_2, 1, 0_3), \\ |e_{12}\rangle &= (1, 0_{15}), |e_{13}\rangle = 1/\sqrt{2}(0_{11}, 1, 0_2, 1, 0), |e_{14}\rangle = 1/\sqrt{5}(0_7, 1, 0_2, \sqrt{3}, 0_2, 1, 0_2), \\ |e_{15}\rangle &= 1/\sqrt{5}(0_2, 1, 0_2, \sqrt{3}, 0_2, 1, 0_7), |e_{16}\rangle = 1/\sqrt{2}(0, 1, 0_2, 1, 0_{11}), \end{aligned}$$

where $0_k \equiv \overbrace{0, 0, \dots, 0}^{k \text{ times}}$.

The biqubit Hamiltonian has the same structure as in the equation (1) with the eigenvalues $-3J, J, -h + J, h + J$.

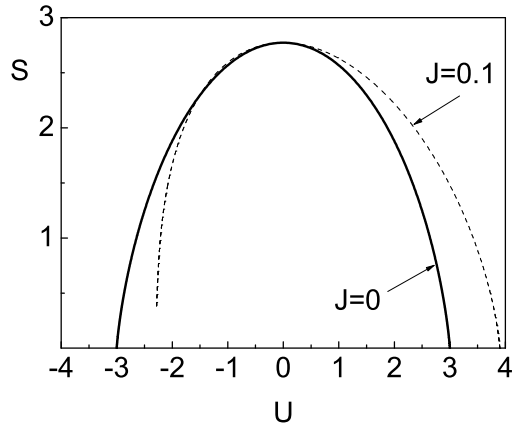


Figure 1. Entropy S is plotted as a function of the internal energy U for a biquartit at the fixed magnetic field $h = 1$.

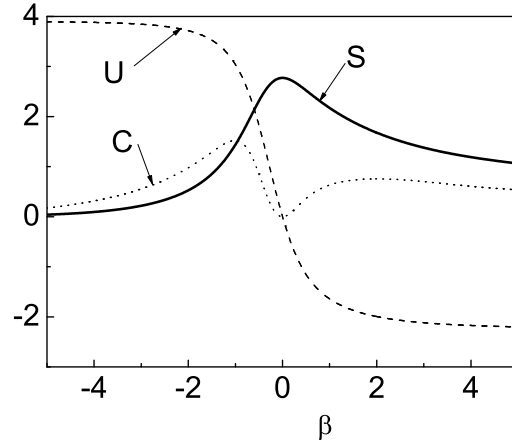


Figure 2. The internal energy U , the entropy S and the heat capacity C are plotted as functions on the inverse temperature β for $h = 1$, $J = 0.1$. At $J = 0$ the entropy and the heat capacity are the even functions, the internal energy is the odd function [15].

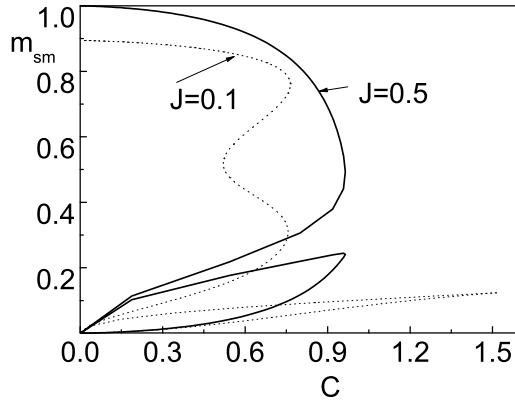


Figure 3. Parametric dependence of the entanglement m_{SM} and the heat capacity C on the inverse temperature at $h = 1$ for different coupling constants. The closed parts of graphs correspond to the negative temperature, and unclosed ones to the positive temperature.

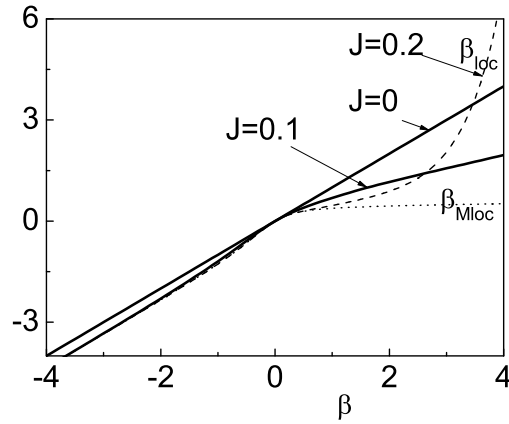


Figure 4. Inverse local temperature of the quartit β_{loc} versus the inverse temperature of the biquartit β for $h = 2$, $J = 0, 0.1, 0.2$.

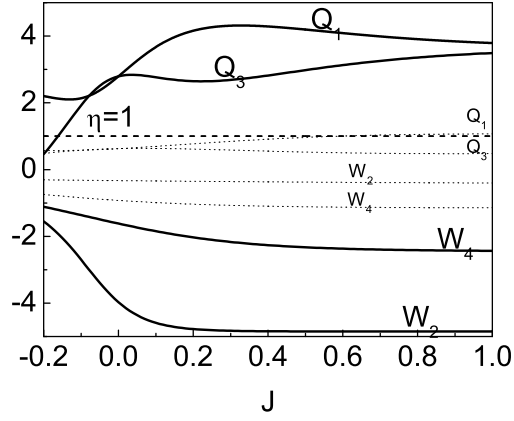


Figure 5. Dependence of heat and work on a coupling constant at all stages of the Otto cycle at $T = -1$, $T' = -3$, $h = 1$, $h' = -1$. The dashed line is the efficiency of the heat energy conversion in work. Hereinafter the bold lines are for the biquartit; the pointwise lines are for the biqubit.

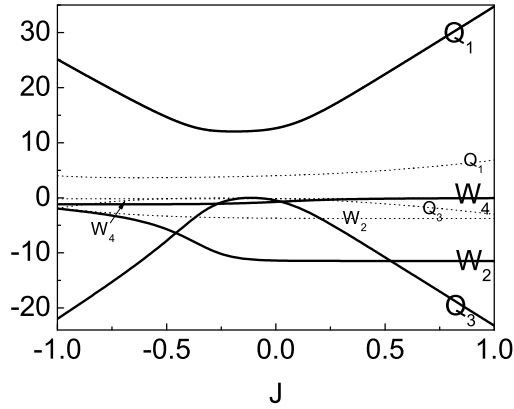


Figure 6. Heat and work versus the coupling constant for $T = -1$, $T' = 2$, $h = 4$, $h' = 0.155$.

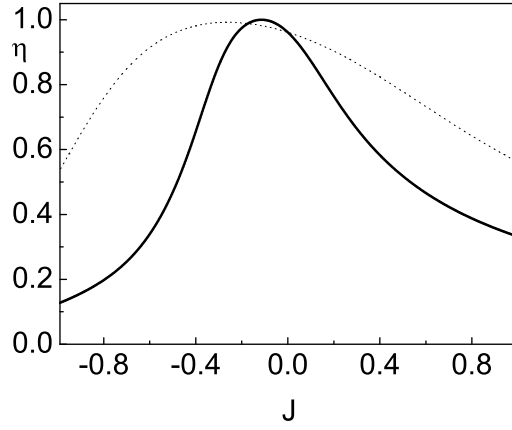


Figure 7. Efficiency of the conversion of heat in work with parameters as in Fig. 6. The heat leakage in the biqubit equals -0.0028 ; the leakage in the biquartit is $Q_3 = -0.0021$. The maximum efficiency equals 0.999 in the biqubit for $J = -0.26$, and in the biquartit for $J = -0.11$.

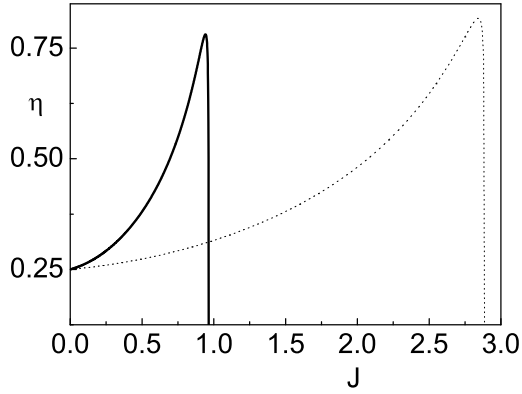


Figure 8. Efficiencies η of the biqubit and the biquartit transformations of heat in work depending on the coupling constant J at $T = 2.5$, $T' = 0.25$, $h = 16$, $h' = 12$. The Carnot limit is 0.9.

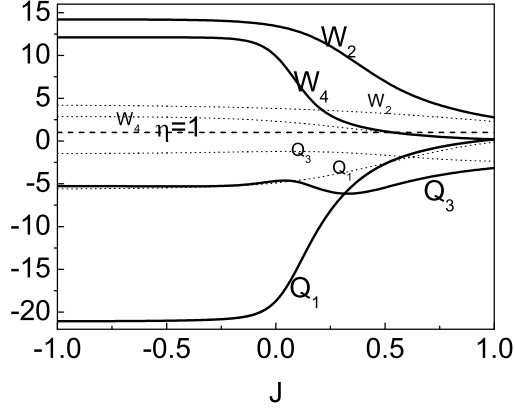


Figure 9. Heat and work versus the coupling constant for $T = 2$, $T' = 1$, $h = 4$, $h' = -1$. The dashed line is the efficiency of conversion of work in the heat energy.

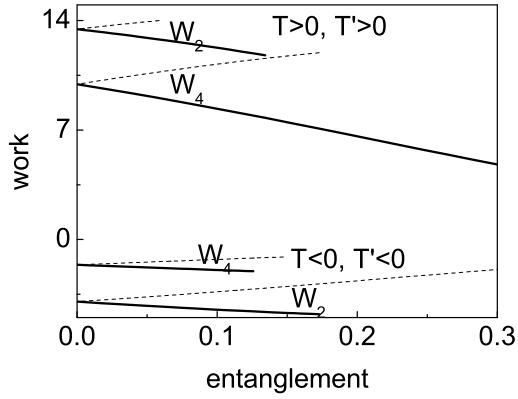


Figure 10. Work $W_2 < 0, W_4 < 0$ versus the entanglement m_{SM} with parameters as in Fig. 5; at $W_2 > 0, W_4 > 0$ parameters as in Fig. 9. Full lines correspond to $J > 0$, dashed ones $J < 0$.

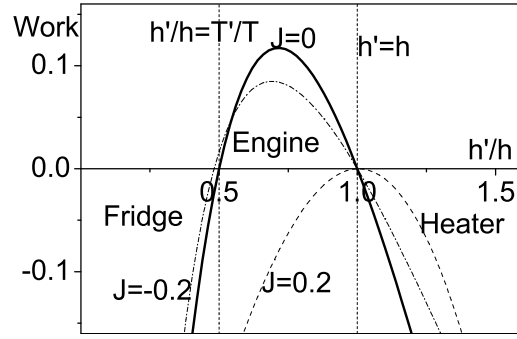


Figure 11. Work versus the control parameter h' at $h = 1, T = 1, T' = 0.5$. Full line correspond to $J = 0$; dashed, dashed-dot ones correspond $J = 0.2, J = -0.2$ respectively.

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